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Analysis of the coupling loss factor between connected plates using the finite element method

Barbosa, R. M.[1]; Medeiros, E. B.[2]

[1,2] Graduate Program in Mechanical Engineering, Federal University of Minas Gerais, Belo Horizonte, MG, Brazil,
rangelmb@yahoo.com.br, bauzer@demec.ufmg.br

Abstract

The coupling loss factor (CLF) of statistical energy analysis (SEA) is employed to characterize the power transmission between L-joined Plates. The aim is to compare different approaches for this parameter and identify factors that influence its results. The wave approach is used to obtain the CLF, and the apparent coupling loss factor (ACLF) is obtained by experimental statistical energy analysis (ESEA) together with the finite element method (FEM). In the procedures, the influence of boundary conditions and damping are analyzed, in which it is found that they do not affect the wave approach. In the ESEA/FEM procedure, the boundary conditions influence more significantly at lower frequencies, and for the higher values of damping used the results approximate the wave approach, which in general tends to occur at higher frequencies. The influence of the applied excitation in FEM is also observed in the ESEA/FEM procedure.

Keywords: Coupling factor, Statistical energy analysis, L-joined Plates, Finite element method

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1. INTRODUCTION

Many human activities involve noise and vibration, as occurs in machinery in industrial processes and vibrations caused in transport vehicles. However, the exposure of the human body to such phenomena can be harmful since they can generate minor discomforts to more aggravated damage to the health, which makes these phenomena an important research subject [1].

In such a research, prediction methods are usually applied due to their practicalities. Among these methods is statistical energy analysis (SEA), which analyzes the energy transmission within a mechanical system by defining parameters that describe the characteristics of this system, such as modal density and coupling loss factor (CLF) [2]. The latter is related to the energy transfer in the system, and its evaluation is one of the most important tasks of the engineering application of SEA [3].

The proper obtention of the CLF can provide accurate predictions through SEA, and proportionate the method as a framework for experimental investigations of the dynamic behavior of systems and in interpreting observations on operating systems [4]. Thus, the study of different approaches to the CLF can be a good way to investigate the proper modeling and application of SEA.

With this purpose, the wave approach and the experimental statistical energy analysis (ESEA) together with the finite element method (FEM) are employed in this work to obtain the coupling factor. Also aiming to evaluate aspects of influence to the coupling factor in the approaches, different boundary conditions and damping values are investigated.

The analyzed system consists of a configuration often found in vehicular and industrial applications, an L-shaped junction between plates. The knowledge of the dynamic behavior of this smaller portion of the structure can conduct to an understanding of the complete structure and also simplify the analysis [5].

2. STATISTICAL ENERGY ANALYSIS

In SEA, the fractions of the system that have similar mode groups are called subsystems. In the SEA model, mechanisms for energy dissipation, excitation by external sources, and energy transmission are assigned to the subsystems [2]. Figure 1 illustrates a generic model.

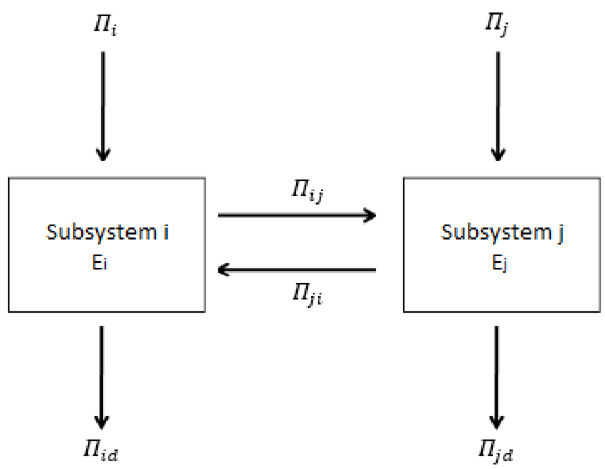


Figure 1: SEA model composed of two subsystems

Figure 1 shows a model composed of two subsystems, in which the energy provided by the external sources (Π_i, Π_j) establishes an energy flow over the model. The balance of energy can be expressed as:

$$\Pi_i + \Pi_{ji} = \Pi_{ij} + \Pi_{id} \, , \tag{1}$$

$$\Pi_j + \Pi_{ij} = \Pi_{ji} + \Pi_{jd} \, , \tag{2}$$

where Π_{ij} and Π_{ji} express the energy flow between the subsystems, calculated as:

$$\Pi_{ij} = \omega \eta_{ij} E_i \, , \tag{3}$$

$$\Pi_{ij} = \omega \eta_{ij} E_i \, , \tag{4}$$

where η_{ij} and η_{ji} are the coupling loss factors and represent the rate of energy exchange between the subsystems, E_i represents the energy of subsystem “i”, and ω the angular frequency.

In Equations (1) and (2), there is also energy dissipation, which is represented by Π_{id} and Π_{jd}, respectively. This energy does not return to the system and can be due to friction or viscosity, for example. It is given by:

$$\Pi_{id} = \omega \eta_{id} E_i \, , \tag{5}$$

$$\Pi_{jd} = \omega \eta_{jd} E_j \, , \tag{6}$$

where η_{id} and η_{jd} are the damping loss factors (DLF) and, in most cases, these are obtained through experimental procedures. When combining Equations (1-6), the power balance equation for two subsystems can be expressed in the matrix form shown in Equation (7) [6].

$$\begin{bmatrix} \Pi_i \\ \Pi_j \end{bmatrix} = \omega \begin{bmatrix} (\eta_{id} + \eta_{ij}) & -\eta_{ji} \\ -\eta_{ij} & (\eta_{jd} + \eta_{ji}) \end{bmatrix} \begin{bmatrix} E_i \\ E_j \end{bmatrix} . \tag{7}$$

3. PROCEDURES

The analyzed plates have the characteristics described in Table 1.

Table 1: Material and geometrical specifications

Width	0.9 m
Length	1.0 m
Thickness	0.002 m
Density	2700 Kg/m³
Poisson’s ratio	0.3296
Young’s Modulus	71 GPa

The plates are connected in an "L" junction configuration by their widths (0.9 m). The junction is continuous (line) with boundary conditions simply supported, which implies that only bending waves are transmitted between the plates.

The results of the analysis are obtained and compared in third-octave bands utilizing the general procedures described in the following.

3.1 Wave approach

The wave transmission across plates junction can be characterized by the transmission coefficient [7]. In SEA, this parameter is also used to determine the coupling loss factor, as expressed in Equation (8).

$$\eta_{12} = \frac{c_g L}{\omega \pi S} \bar{\tau}_{12} \,, \tag{8}$$

where c_g is the group speed, L is the length of the junction, S is the plate area, and $\bar{\tau}_{12}$ is the angular-average transmission coefficient. Assuming diffuse incidence:

$$\bar{\tau}_{12} = \int_0^{\pi/2} \tau_{12}(\theta) \cos\theta \, d\theta \,. \tag{9}$$

The latter can be obtained by the wave approach, in which the vibrational fields are modeled as a superposition of traveling waves in semi-infinite plates. In this process, the energy transferred between the plates is analyzed by establishing continuity and equilibrium conditions at the junction, which makes it possible to relate the incident, transmitted, and reflected waves [6].

Applying this approach, the amplitudes (α) of transmitted and reflected waves are obtained and applied to calculate the wave power as follows [7]:

$$P_B = \left(\frac{\rho h \omega^3 \alpha^2}{k_B}\right) \cos\theta \,, \tag{10}$$

where θ is the wave heading, k_B is the bending wave number, h and ρ are thickness and density of the plate, respectively.

The transmission coefficient will be the ratio between the incident wave power (${P_i}^1$) and the transmitted wave power (${P_j}^2$), expressed by:

$$\tau_{pr}^{ij}(\omega, \theta) = {P_j}^2 / {P_i}^1 \,. \tag{11}$$

Calculating the angular-average transmission coefficient from Equation (9), the coupling factor can be obtained from Equation (8).

3.2 Experimental statistical energy analysis

Experimental statistical energy analysis (ESEA) can be used to determine the coupling factor in alternative to the analytical method, especially when the last is led to wrong values due to the complexity of the subsystems and the coupling junctions [8]. The ESEA matrix form is given in Equation (12), which is derived from the energy balance equations of SEA.

$$\begin{bmatrix} \sum_{n=1}^{N} \eta_{1n} & -\eta_{21} & \cdots & -\eta_{N1} \\ -\eta_{12} & \sum_{n=1}^{N} \eta_{2n} & & \vdots \\ \vdots & & \ddots & \\ -\eta_{1n} & \cdots & & \sum_{n=1}^{N} \eta_{Nn} \end{bmatrix} \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1N} \\ E_{21} & E_{22} & & \vdots \\ \vdots & & \ddots & \\ E_{N1} & \cdots & & E_{NN} \end{bmatrix} = \begin{bmatrix} \Pi_{in,1}/\omega & 0 & \cdots & 0 \\ 0 & \Pi_{in,2}/\omega & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & & \Pi_{in,N}/\omega \end{bmatrix} . \tag{12}$$

In the method, the data can be acquired either by laboratory or numerical procedures, in which the finite element method (FEM) is commonly applied since some difficulties can be avoided, reducing costs and numerical processing time of the experimental implementation [9].

The approach involves excitation of each subsystem at a time and obtaining the displacement responses of the elements in FEM, these are used to determine the vibrational energy of the structure as follows [10]:

$$E = \frac{1}{2} \omega^2 \sum_{n=1}^{N} m_n {\eta_n}^2 \,. \tag{13}$$

In Equation (13), η_n is the out-of-plane displacement of the element and m_n is the mass of the element. The input of energy generated by the force application on the nodes is given by [8]:

$$\Pi_{in} = \frac{\omega}{2} \sum_{n=1}^{N} (Im\{F\} * Re\{\eta_{nó}\} - Re\{F\} * Im\{\eta_{nó}\})_n \,, \tag{14}$$

where F is the applied force and η_{n6} is the resulting displacement at the node.

Using the generated results as input in Equation (12), the coupling factors can be obtained by inverting the matrix. The coupling factor obtained through the deterministic approach is usually called the apparent coupling loss factor (ACLF), to distinguish it from the coupling loss factor (CLF) obtained by the wave approach [11].

3.3 Numerical experiments

The FEM model from the software used in the research (ANSYS) is illustrated in Figure 2. The element size is less than 1/6 of the shortest wavelength of the analysis and the boundary condition of the junction is simply supported, which allows only bending waves to be transmitted.

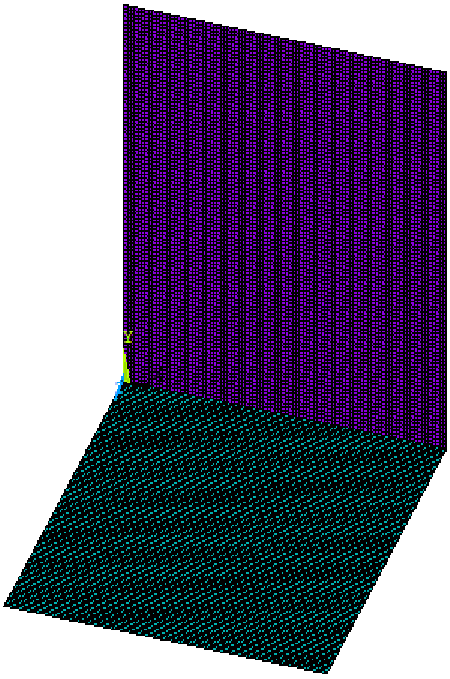


Figure 2: FEM model

In the FEM model, the excitation is performed by rain-on-the-roof (ROTR), which can enable equal excitation of local modes of the structure [8, 12]. The adequacy of the FEM analysis regarding the mesh size is also verified by the mesh error proposed by Hopkins [13]. The maximum error found in the analysis was near 10%, which is less than the recommended 40% and represents adequacy.

Within the analysis, variations in the results are evaluated by changing the DLF values (0.02, 0.04, 0.08, 0.16) and the boundary conditions for the edges of the plates (free or simply supported), with exception to the junction which is always simply supported. The data are obtained in 10 Hz intervals and averaged values are calculated for third-octave bands (100-3150) Hz.

4. RESULTS AND DISCUSSION

The results obtained by applying the proposed procedures and the corresponding analyses are presented in this section.

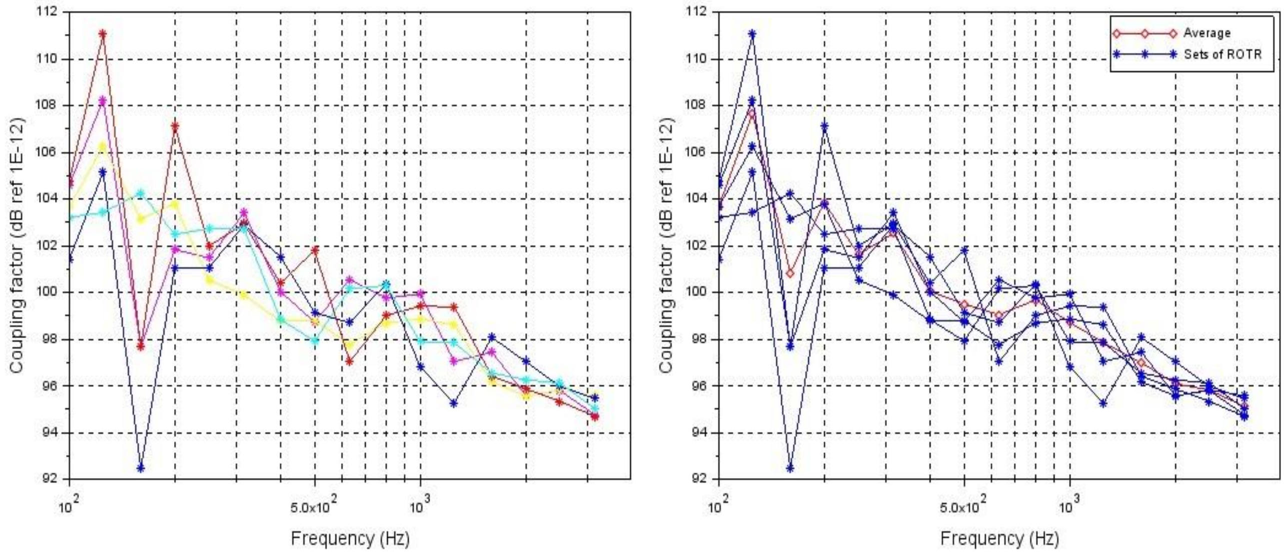
4.1 Excitation

The force applied to the FEM model by means of the ROTR corresponds to complex forces acting on all nodes, except those near and at the edges of the plate, having unit magnitude and different from each other by the phase values. Figure 3 (a) shows ACLF results for 5 different sets of ROTR applied to the model.

This first analysis was based on the reported by [8, 11], which presents the applied force as an influencing factor to the ACLF results in the employed procedure. The sensitivity of ACLF results to the definition of excitation is noticeable comparing the curves in Figure 3 (a). This evidences that the use of only one set of ROTR could lead to inconclusive results.

For this reason, it becomes inductive to obtain average values of ACLF from the results of ROTR sets. This procedure is shown in Figure 3 (b).

As expected, average values reduce excessive oscillations of the results and appear to represent the system more adequately. For this reason, this same procedure for averaging is adopted in the following analyses, using the same 5 sets of ROTR excitation.



(a) 5 Sets of ROTR

(b) Average of 5 Sets of ROTR

Figure 3: Simply supported edges, DLF=0.04

4.2 Boundary conditions

The influence of the boundary conditions is analyzed by changing the displacement constraints of the plate edges and proceeding with ESEA. The averaged ACLF values for the models with edges free and with edges simply supported are presented in Figure 4, as well as values obtained by the wave approach.

Figure 4 shows the most pronounced influence of the boundary conditions in the lower frequency ranges, where the differences between the curves for the conditions simply supported and free edges are larger. Increasing the frequency is verified the approximation between these ACLF results, which represent less influence of the boundary conditions in the higher frequency ranges. Increasing the

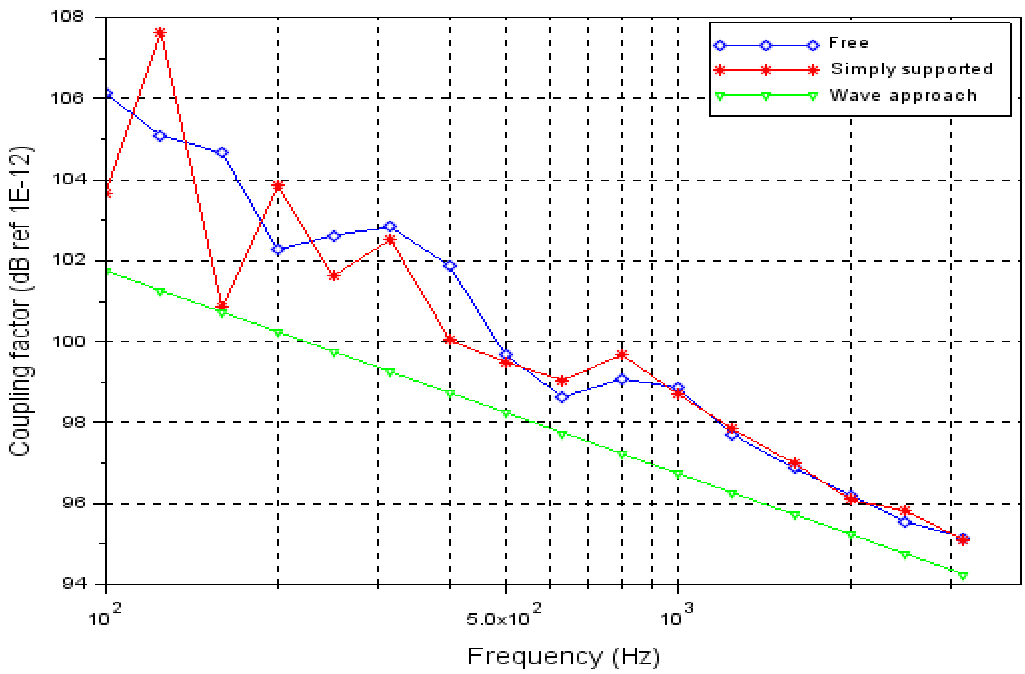


Figure 4: Comparison between ESEA (simply supported and free edges, DLF=0.04) and the wave approach

frequency, there is also a trend of both curves to approximate the wave approach.

At lower frequencies, the structure response tends to be guided by global modes, which are modified by the boundary conditions. This behavior may be the reason for the greater influence of the boundary conditions in the frequency range observed in Figure 4. On the other hand, with increasing the frequency, the structure response tends to be guided by local modes, which may be associated with the decrease of the influence of the boundary conditions [9].

4.3 Damping loss factor

In the wave approach, damping is not directly treated in obtaining the coupling factor, whereas in ESEA it can be considered in the definition of the FEM model. In order to compare the effects of these aspects, the results by using both methods are shown in Figure 5.

The ESEA results correspond to 4 distinct DLF values, which present curves with similar oscillation behaviors throughout the frequency range. It is also noticeable in the low frequency region that these curves have a greater distance from the wave approach. This behavior was also described in [11], which suggests that in this region considerations used in SEA subsystems may be inappropriate.

In Figure 5, it is also verified the greater proximity of the wave approach to the highest damping values adopted in ESEA, which makes the curve of η = 0.16 the closest to that approach. This also illustrates the damping influence on energy transmission between subsystems in the deterministic analysis. Another aspect of these results concerns the tendency of approximation of the ESEA curves to the results of the wave approach with the increase of frequency, as also has occurred in previous analysis.

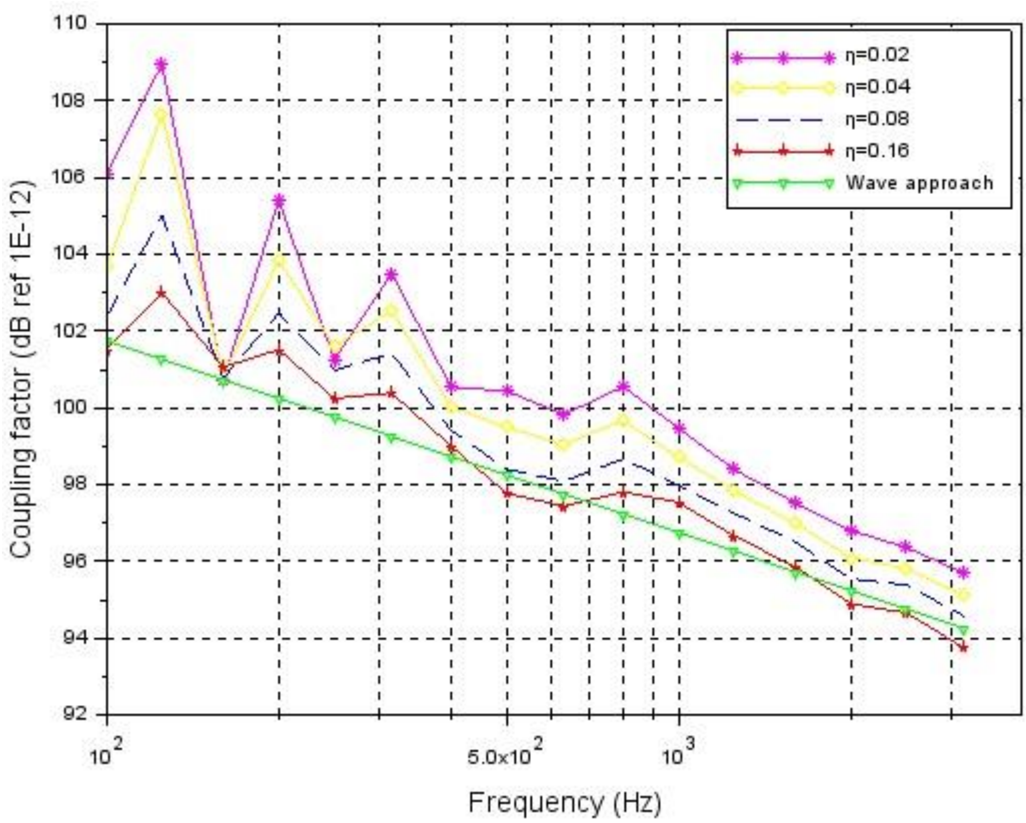


Figure 5: Comparison between ESEA (different DLF, simply supported) and the wave approach

5. CONCLUSION

The wave approach and a procedure with ESEA and FEM were used to obtain the coupling factor of an L-joined plates system. The junction was continuous and assumed to transmit only bending waves.

In the procedures, the influence of the damping loss factor and the boundary conditions have been analyzed, of which the wave approach has been found independent. In FEM, these parameters lead to modifications of the model, such as the dynamic behavior, natural frequencies, and modal shapes. When using FEM results as input in ESEA, that reflects on the coupling values.

As consequence, the boundary conditions influence in ESEA has been observed, especially at lower frequencies. This influence tends not to be relevant at higher frequency ranges. In the damping analysis, the higher damping values used have led to greater proximity between the ESEA and the wave approach results, which may represent complications in the latter for structures with low damping, especially at low frequencies.

In general, the results of the approaches approximate with frequency increase. In the ESEA with FEM procedure, it has also been observed the excitation influence, which makes it an important parameter to be analyzed.

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